Direct measurement of chirp parameters of high-speed Mach-Zehnder-type optical modulators

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Abstract

A method is described for measuring the chirp parameters and optical responses of Mach-Zehnder-type optical modulators by using an optical spectrum analyzer. The chirp parameter and the optical response for small-signal operation are expressed by the magnitude of the phase induced in each optical path of the Mach-Zehnder waveguides. The induced phase can be obtained from the ratio of the high-order optical harmonic intensities generated by large-signal operation in the millimeter-wave region. This method permits the chirp parameter to be evaluated at a specific frequency, while it is assumed to be independent of the frequency of the applied electric rf signal in the conventional method. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Chirping, the parasitic phase modulation on an intensity optical modulation, affects the transmission performance of high-speed and long-haul optical transmission systems. Optical modulators using Mach-Zehnder waveguide structures can reduce the chirping in high-speed intensity-modulated lightwaves, so they have important application in optical and wireless telecommunications, such as broad-band optical modulation up to 40 GHz for trunk lines [1-3], and narrow-band optical modulation for fiber-radio systems [4,5]. If the rf electric fields applied to the optical paths of the Mach-Zehnder structure are symmetric, we can obtain a pure intensity modulation without phase modulation. However, most commercial modulators use asymmetric structures to reduce the halfwave voltage ($V_\pi$), so they suffer chirping. Consequently, to develop high-speed optical modulators using Mach-Zehnder structures, a method for exact measurement of the chirping in the millimeter-wave region is required. The chirp parameter of a modulator, which is defined by the ratio between the phase and intensity modulations, can be easily measured using dispersive media such as optical fibers [6]. However, in the conventional method the chirp parameter is
assumed to be independent of the frequency of the applied electric signal, while it may actually depend on the frequency, especially in the millimeter-wave region. In this paper, we introduce a chirp measurement method that uses the spectrum of the output lightwave directly. The chirp parameter at a specific frequency can be precisely measured from the ratio of the high-order optical harmonic intensities, while conventional methods using the optical spectrum cannot [7,8].

2. Chirp parameter of Mach-Zehnder optical modulator

Consider a lightwave output from a Mach-Zehnder optical modulator whose electric field is described by

\[ E \exp(i\phi + i\omega_0 t) = \frac{1}{2} \left\{ \exp(iA_1 \sin \omega_m t + i\phi_{B1}) + \exp(iA_2 \sin \omega_m t + i\phi_{B2}) \right\} \times \exp(i\omega_0 t), \]  

(1)

\[ E \equiv \cos(\Delta \phi), \]  

(2)

\[ \Delta \phi \equiv \frac{1}{2} \{ (A_1 \sin \omega_m t + \phi_{B1}) - (A_2 \sin \omega_m t + \phi_{B2}) \}, \]  

(3)

\[ \phi \equiv \frac{1}{2} \{ (A_1 \sin \omega_m t + \phi_{B1}) + (A_2 \sin \omega_m t + \phi_{B2}) \}. \]  

(4)

\( E \) is the amplitude of the electric field. \( \phi \) is the phase retardation induced by the modulator. \( A_1 \) and \( A_2 \) denote the magnitude of the optical phase induced by the rf electric field applied to each path of the Mach-Zehnder structure. \( \phi_{B1} \) and \( \phi_{B2} \) are the phase delays due to the differences in optical lengths between paths. \( \omega_m \) is the angular frequency of the electric signal fed to the modulator, and \( \omega_0 \) is that of an unmodulated lightwave from the source.

The chirp parameter, the ratio of the amplitude modulation and the phase modulation, is defined by [9]

\[ \alpha = \frac{d\phi}{dt} = \frac{1}{E} \frac{dE}{dt}. \]  

(5)

Thus, we can obtain

\[ \alpha = -\cot(\Delta \phi) \frac{A_1 + A_2}{A_1 - A_2}. \]  

(6)

Consider the case of a small amplitude modulation where \( A_1, A_2 \ll 1 \). \( \phi_{B} (\equiv \phi_{B1} - \phi_{B2}) \) can be controlled by applying dc voltage to the electric port of the modulator. If we assume \( \phi_{B} = -\pi/2 \), which corresponds to an optimal condition for small-amplitude modulation, the chirp parameter can be expressed in terms of \( A_1 \) and \( A_2 \):

\[ \alpha \simeq \alpha_0 \equiv \frac{A_1 + A_2}{A_1 - A_2}. \]  

(7)

In the push-pull configuration, the polarity of \( A_1 \) is usually opposite that of \( A_2 \). If the electrode of the modulator is symmetric with respect to the cross-section of the optical waveguide, as shown in Fig. 1, \( A_1 \) equals \(-A_2 \), so \( \alpha_0 \) equals 0, which corresponds to a zero-chirp modulator. If we consider that \( A_2 \) equals zero, \( \alpha_0 = 1 \), which corresponds to the case where the electric signal is applied to only one of the optical paths. On the other hand, \( \alpha_0 \) for the case \( A_1 = A_2 \) goes to infinity, which corresponds to pure phase modulation. Assuming that the nonlinear optical effects except the Pockels effect are negligible, the ratio between \( A_1 \) and \( A_2 \) does not depend on the intensity of the electric signal. Thus, \( \alpha_0 \) is also an intrinsic parameter of the modulator. In this paper, we consider the measurement of the chirp parameter \( \alpha_0 \) and the optical response of a Mach-Zehnder-type optical intensity modulators whose cross-section is asymmetric, as shown in Fig. 2. Without losing any generality, we can assume that \( |A_1| \) is greater than \( |A_2| \), that \( A_1 \) is positive, and that \( A_2 \) is negative, so \( 0 < \alpha_0 < 1 \).
3. Principle

The optical spectrum of the lightwave output from the modulator is a function of $A_1$, $A_2$ and $\phi_B$ ($= \phi_{B1} - \phi_{B2}$). The ratio between the $(n+1)$th and $n$th order harmonic intensities in the spectrum is expressed by

$$R_n = \frac{|J_n(A_1) + J_n(A_2) \exp(i\phi_B)|^2}{|J_{n+1}(A_1) + J_{n+1}(A_2) \exp(i\phi_B)|^2} \tag{8}$$

$$= \frac{\{J_n(A_1)\}^2 + \{J_n(A_2)\}^2 + 2J_n(A_1)J_n(A_2) \cos \phi_B}{\{J_{n+1}(A_1)\}^2 + \{J_{n+1}(A_2)\}^2 + 2J_{n+1}(A_1)J_{n+1}(A_2) \cos \phi_B} \tag{9}$$

If some $R_n$s are known, $A_1$, $A_2$, and $\phi_B$ can be derived from simultaneous transcendental equations. In general, the number of equations to be solved is equal to the number of unknown variables. For example, if $A_1$ and $A_2$ are unknown, they can be obtained from two equations of $R_0$ and $R_1$. Because these equations are transcendental, several solutions may be derived, and some of them may be unphysical. Actual solutions can be obtained by using more equations than the number of unknown variables. Thus, higher-order harmonic intensities in the optical spectrum should be measured precisely to obtain $A_1$, $A_2$, and $\phi_B$.

Factor $\cos \phi_B$ in Eq. (9) shows the connection between the optical spectrum and the dc-bias voltage. $\phi_B$ depends on the environmental conditions, which is known as dc-drift [10-13]. Because the halfwave voltage $V_\pi$ does not change much, the effect of dc-drift can be eliminated by sweeping the dc-bias voltage across two times $V_\pi$ for dc, which corresponds to a period of $\cos \phi_B$. The ratio of the optical harmonic intensities is expressed by

$$R_n = \frac{\{J_n(A_1)\}^2 + \{J_n(A_2)\}^2}{\{J_{n+1}(A_1)\}^2 + \{J_{n+1}(A_2)\}^2}, \tag{10}$$

and does not depend on the dc-bias voltage, so $A_1$ and $A_2$ can be precisely determined. In addition, $\alpha_0$ depends only on the ratio of $A_1$ and $A_2$, so it is independent of the rf power applied to the modulator. Consequently, the effects of rf-power fluctuation and dc-drift can be eliminated in the measurement of the chirp parameter.

4. Experimental results

The experimental setup we used is shown in Fig. 3. The modulator tested was a Mach-Zehnder-type LiNbO$_3$ traveling-wave optical-intensity modulator. Its electrode had an asymmetric cross-section, as shown in Fig. 2. A large rf electric signal amplified by a traveling-wave tube amplifier was fed to an optical modulator to generate higher order optical harmonics. The ratio of the harmonic intensities was measured using an optical spectrum analyzer. One of the optical spectrums

![Fig. 2. Cross-section of asymmetric intensity modulator.](image)

![Fig. 3. Setup for measurement of chirp parameters and optical responses.](image)
used to obtain the parameters $A_1$, $A_2$ and $\phi_B$ is shown in Fig. 4. The $\cos \phi_B$ obtained from Eq. (9) is shown in Fig. 5 as a function of the dc-bias voltage. It approximately follows a sinusoidal curve with period equal to twice the halfwave voltage for dc, $V_{\pi\text{(dc)}}$, which is estimated to be 4.9 V, and is in fair agreement with the value obtained by usual measurement of dc-voltage versus optical output power.

In large-signal operation, the dc-drift due to temperature fluctuation can be larger than $\pi$. To eliminate the effect of the dc-drift, the dc-bias voltage was swept with an amplitude of $2V_{\pi\text{(dc)}}$ by using a function generator. The period of the sweeping should be much smaller than that of the optical spectrum measurement, but should not be so small as to pass the low-pass filter in the bias feeding circuit shown in Fig. 3. The output of the function generator is a triangular wave whose frequency and amplitude are 100 kHZ and 9.56 V. Using Eq. (10), enables $A_1$ and $A_2$ to be obtained from the ratio of the optical harmonics measured by the optical spectrum analyzer.

Fig. 6 shows induced phases $A_1$ and $A_2$ as functions of the voltage (zero-to-peak) of the applied rf electric signal $V_{rf}$ which had a frequency of 30 GHz. Reflecting the characteristics of the Pockels effect, $A_1$ and $A_2$ Were proportional to $V_{rf}$, so that the higher order nonlinear effects were negligibly small. In contrast, as shown in Fig. 7, the chirp parameter $\alpha_0$ did not depend on the rf voltage and was $0.786 \pm 0.003$. Consequently, the fluctuations in the rf voltage does not affect the measurement of $\alpha_0$.

Halfwave voltage $V_{\pi}$ was derived from $A_1$ and $A_2$ by using $V_{\pi} = \pi V_{rf} (A_1 - A_2)$, and is shown in Fig. 8 as a function of the rf frequency, where $V_{rf}$ was 14.6 V. $V_{\pi}$ was much larger than $V_{\pi\text{(dc)}}$ and increased with respect to the frequency due to the loss in the electrode and the velocity mismatch between the rf electric signal and the lightwave.
We calculated the chirp parameter from the measured optical spectrums, as a function of the rf frequency in the regions of 10-18 GHz and of 28-39 GHz, where $V_{	ext{rf}}$ was 9.0 V at 10-18 GHz and was 14.6 V at 28-39 GHz. As shown in Fig. 9, it increased with the frequency, which means that the ratio of $A_1$ and $A_2$ depends on the frequency. The standard deviation from the fitting line was 0.017.

If the TEM analysis, where the electro-magnetic fields along the propagation direction are assumed to be zero [14], gives a good approximation of the electric field of the electrode, the cross-section of the electric field pattern does not depend on the rf frequency. In a high-frequency region such as 30-40 GHz, the asymmetry of the electric fields at each optical path becomes so large that the TEM approximation does not give precise electric field patterns. The chirp parameter measured using the conventional method at a region of 5-27 GHz [6] was $\alpha_0 = 0.71 \pm 0.04$, which is in good agreement with the results obtained by the proposed method shown in Fig. 9. The measurement error in the conventional method is larger than in our proposed method, because the dependence of $\alpha_0$ on the rf frequency is neglected in the conventional method.

5. Conclusion

We have proposed a method for measuring the chirp parameters and optical responses of Mach-Zehnder-type optical modulators. In this method, we directly obtain the induced phase at each optical path of the modulator from the ratio of the high-order optical harmonic intensities by using an expression consisting of Bessel functions. To achieve precise measurement, the effect of dc-drift is eliminated by sweeping the dc-bias voltage. The halfwave voltage and the chirp parameter were calculated from the induced phases. The chirp parameter of the tested modulator was $0.695 \pm 0.017$ at 10 GHz, and $0.797 \pm 0.017$ at 39 GHz, while the parameter obtained by the conventional method was $0.71 \pm 0.04$. This demonstrates the effectiveness of our proposed method.
method was $0.71 \pm 0.04$ at 5-27 GHz. The dependence on the rf frequency cannot be explained by the TEM analysis.

References


